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Adiabatic and isocurvature fluctuations of Affleck-Dine field in D-term inflation model

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Abstract

We reconsider fluctuations of Affleck-Dine (AD) field in a D-term inflation model. Contrary to the previous analysis, we find that the spectrum of the adiabatic fluctuations is almost scale invariant even if the AD field has a large initial value. Furthermore, we study the isocurvature fluctuations of the AD field and estimate the ratio of the isocurvature to adiabatic power spectrum. The dynamics of the inflaton and AD fields sets the upper bound for the value of the AD field, leading to a lower limit for isocurvature perturbation. It is shown that the recent Cosmic Microwave Background data give a constraint on the D-term inflation and the AD field.

1 Introduction

Minimal Supersymmetric Standard Model (MSSM) have many flat directions, along which there are no classical potentials. The flat directions are only lifted by soft SUSY breaking mass terms and non-renormalizable terms. Such flat directions have several interesting consequences in cosmology such as baryogenesis and Q-ball formation. In Affleck-Dine mechanism for baryogenesis, a complex field along a flat direction (AD field) has a large field value during inflation. After inflation the AD field starts to oscillate when its mass becomes larger than the Hubble parameter H . At that time the AD field obtains velocity in the phase direction because of the baryon number violating operator, and the baryon number is generated very efficiently.

For inflation models in which the accelerated cosmic expansion is caused by a F-term potential, the AD field generally obtains an effective mass of the order of H through supergravity effects during inflation. In this case, the expectation value of the AD field Φ is determined by the balance between mass term $cH^2|\Phi|^2$ and the non-renormalizable term where c is order one constant. If the mass term is positive ($c > 0$) as expected from minimal supergravity, the potential takes minimum at $\Phi = 0$ and the AD mechanism does not work. Thus the supergravity effects must lead to negative mass term (i. e. $c < 0$) for baryogenesis in F-term inflation models.

Supersymmetric inflation models are also constructed with use of D-term potential. In the D-term inflation models supergravity effects do not induce $O(H^2)$ mass terms and the AD field has only soft SUSY breaking mass term of the order of weak scale. Thus, the AD field can have a large expectation value during inflation, which makes the AD baryogenesis work.

Since the effective masses of the AD field in D-term inflation models are much smaller than the Hubble parameter during inflation, the AD field has large fluctuations which may give some contribution to the density fluctuations of the universe. In the previous work [1, 2], the fluctuations of AD field was considered and it was pointed out that the the fluctuations of the AD field change the spectral index of the adiabatic fluctuations produced in the D-term inflation, from which the expectation value of the AD field is constrained. The authors in Refs. [1, 2] also pointed out that the small expectation value of the AD field during inflation leads to lager isocurvature (baryon) perturbations which are induced by the fluctuations in the phase direction of the AD field. However, their estimation of the adiabatic fluctuations was too naive, and one needs careful treatment for the multi-field dynamics.

In this paper, we reconsider the AD field fluctuations in the D-term inflation model. We find that the spectrum of the adiabatic fluctuations is almost scale invariant even if the AD field has a large initial value. Furthermore, we study the isocurvature fluctuations of the AD field and estimate the ratio of the isocurvature to adiabatic power spectrum. It is shown that the recent Cosmic Microwave Background (CMB) data give a constraint on the model parameters of D-term inflation and the AD field.

2 Dynamics of AD and inflaton fields

Let us discuss the dynamics of the AD and inflaton fields in the D-term inflation model. The flat direction corresponding to the AD field can be lifted by soft SUSY breaking mass term and the non-renormalizable term coming from the superpotential given by

$$W(\Phi) = \frac{\lambda\Phi^d}{dM^{d-3}}, \quad (1)$$

where $M \equiv \frac{1}{\sqrt{8\pi G}}$ is the reduced Planck mass, λ is an $O(1)$ coupling constant, and $\Phi \equiv \frac{\phi e^{i\theta}}{\sqrt{2}}$ is the AD field. Then, the potential is written as [3]

$$V(\phi) = m^2|\Phi|^2 + \left(\frac{am_{3/2}\Phi^d}{dM^{d-3}} + h.c. \right) + \frac{\lambda^2|\Phi|^{2(d-1)}}{M^{2(d-3)}}, \quad (2)$$

where m is the soft mass, $m_{3/2}$ is the mass of gravitino, and a is an $O(1)$ complex constant. The dimension d is an even number, and the cases of $d = 4, 6$ are considered in this paper. Note that the AD scalar ϕ should be less than $O(M)$, otherwise the potential cannot be described as Eq. (2)

The tree-level scalar potential for D-term inflation is given by [4]

$$V = |\kappa|^2 \left(|\psi_+\psi_-|^2 + |S\psi_+|^2 + |S\psi_-|^2 \right) + \frac{g^2}{2} \left(|\psi_+|^2 - |\psi_-|^2 + \xi^2 \right)^2, \quad (3)$$

where κ is the coupling constant of the interaction between S and ψ_\pm , ξ^2 is the Fayet-Illiopoulos term, and g is the $U(1)_{FI}$ gauge coupling. Though the global minimum of this potential is at $S = \psi_+ = 0$, $|\psi_-| = \xi$, the local minimum is at $\psi_+ = \psi_- = 0$ for $|S| > S_c \equiv g\xi/\kappa$. We can take S real using the $U(1)_{FI}$ phase rotation. With use of $\sigma \equiv \sqrt{2}S$, for $\sigma > \sigma_c = \sqrt{2}g\xi/\kappa$, the potential of σ up to a 1-loop correction is given by [4]

$$V(\sigma) = V_0 + \frac{g^4\xi^4}{16\pi^2} \ln \left(\frac{\sigma}{Q} \right), \quad (4)$$

where $V_0 = \frac{g^2\xi^4}{2}$, and $Q = \sigma_c$ is a renormalization point. If the potential is dominated by $V(\sigma)$, σ is related to the number of e-folds (of inflation) N ,

$$\sigma \simeq \frac{gM}{2\pi} \sqrt{N}. \quad (5)$$

Assuming that the present scale crosses the Hubble radius at $N = 50$ during inflation, the COBE normalization ($V^{\frac{3}{2}}/V' = 5.3 \times 10^{-4}$) fixes

$$\xi = 7.05 \times 10^{15} \text{ GeV}. \quad (6)$$

If ϕ starts with $\phi_i \sim O(M)$, the potential is dominated by the F-term of the AD field. Then there arises $O(H^2)$ mass term for the inflaton due to supergravity

effects, and it rapidly rolls down to its true minimum. Therefore inflation does not occur at all in this case. In order to have a successful inflationary model, ϕ_i must be less than ϕ_c , which is given by

$$\phi_c = \sqrt{2} \left(\frac{g}{\sqrt{2}\lambda} \xi^2 M^{d-3} \right)^{\frac{1}{d-1}}. \quad (7)$$

For $\phi_i \leq \phi_c$, the universe is dominated by the D-term of the inflaton field.¹ Thus the D-term driving inflation accounts for the necessary $O(50)$ e-folds. This gives $\sigma_i = O(gM)$. Then it is easy to show that the slow roll condition for σ is satisfied,

$$\epsilon_\sigma \ll |\eta_\sigma| \simeq M^2 \frac{V(\sigma)''}{V(\sigma)} \simeq \frac{1}{2N} \ll 1, \quad (8)$$

where ϵ_σ and η_σ are slow roll parameters [5]. Once the inflation begins, the AD field rapidly oscillates with a decreasing amplitude. When the amplitude of the oscillation becomes small enough ($\phi \lesssim \phi_{sr}$), ϕ begins to slow-roll. ϕ_{sr} is obtained by solving $\eta_\phi(\phi_{sr}) = M^2 \frac{V(\phi)''}{V(\sigma)} \simeq 1$:

$$\phi_{sr} = \left(\frac{2^{d-1} M^{2d-8} V_0}{(2d-2)(2d-3)\lambda^2} \right)^{\frac{1}{2d-4}}, \quad (9)$$

$$= \begin{cases} \left(\frac{2}{15} \right)^{\frac{1}{4}} \sqrt{\frac{g}{\lambda}} \xi & d = 4 \\ \left(\frac{8}{45} \right)^{\frac{1}{8}} \left(\frac{g}{\lambda} \right)^{\frac{1}{4}} \sqrt{\xi M} & d = 6 \end{cases}. \quad (10)$$

Therefore, the expectation value of the AD field ϕ at the horizon exit of the present scale is generally less than $O(\phi_{sr})$. Thus, we have an upper limit to ϕ ,

$$\phi \lesssim \phi_{sr}. \quad (11)$$

Note that this upper limit to the AD field value is totally due to its dynamical property and the requirement that an inflation should occur, not any observational constraint.

3 Adiabatic Fluctuations

Next, we calculate the fluctuations of the AD field ϕ and inflaton σ . According to Ref. [6], the gauge-invariant curvature perturbation Φ_H is given by

$$\Phi_H = -C_1 \frac{\dot{H}}{H} + \frac{C_3}{3V_{total}^2} \left(V'(\sigma)^2 V(\phi) - V(\sigma) V'(\phi)^2 \right), \quad (12)$$

¹ Precisely speaking, the inflation can occur even if $\phi_i \gtrsim \phi_c$ (say, $\phi_i = 1.5\phi_c$, $\sigma_i = M$). In any case, the AD field value is set below ϕ_{sr} along the same argument, once the D-term driving inflation begins.

where²

$$C_1 = \frac{H}{V_{total}} \left(V(\sigma) \frac{\delta\sigma}{\dot{\sigma}} + V(\phi) \frac{\delta\phi}{\dot{\phi}} \right), \quad (13)$$

is the growing adiabatic mode, and

$$C_3 = \frac{1}{2H} \left(\frac{\delta\sigma}{\dot{\sigma}} - \frac{\delta\phi}{\dot{\phi}} \right), \quad (14)$$

is the isocurvature mode. For $\phi < \phi_{sr}$, V_{total} is dominated by V_0 , and $\dot{\sigma} \leq \dot{\phi}$. Hence $C_1 \simeq H \frac{\delta\sigma}{\dot{\sigma}}$. In other words, the main contribution to the adiabatic fluctuation comes from the inflaton. Therefore the primordial spectrum is almost scale-invariant as usual. With this approximation, the primordial spectrum is given by

$$k^{\frac{3}{2}} \Phi_H(k) \simeq \frac{\sqrt{12}\pi}{5} \left(\frac{\xi}{M} \right)^2 \sqrt{50 - \ln \frac{k}{k_0}}, \quad (15)$$

where $k_0^{-1} \equiv 3000 h^{-1} \text{Mpc}$.

In order to check the above estimate, we also solve the evolution of zero modes and fluctuations of the AD and inflaton fields and the gauge-invariant curvature perturbation Φ_H by numerical calculation, following Ref.[7]. We present the evolution equations in synchronous gauge as [7]

$$\kappa \equiv \left(\frac{k^2}{a^2} \mathcal{R} - 4\pi G \delta\rho \right) / H, \quad (16)$$

$$\delta\rho_{com} \equiv \delta\rho - 3H\Psi, \quad (17)$$

$$\dot{\mathcal{R}} = 4\pi G \Psi, \quad (18)$$

$$\delta\dot{\rho}_{com} = -3H \left(1 + \frac{1}{2}(1 + \omega) \right) \delta\rho_{com} + \frac{k^2}{a^2} \left(\frac{(\rho + p)\mathcal{R}}{H} + \Psi \right), \quad (19)$$

$$\dot{\Psi} = -3H\Psi - \delta\rho, \quad (20)$$

$$\delta\dot{p} = \frac{1}{3}\delta\rho_{com} + H\Psi + \frac{2}{3} \sum_j \left(\dot{\phi}_j \delta\phi_j - 2 \frac{\partial V_{total}}{\partial \phi_j} \delta\phi_j \right), \quad (21)$$

$$\ddot{\delta\phi}_j + 3H\dot{\delta\phi}_j + \frac{k^2}{a^2} \delta\phi_j + \sum_i \frac{\partial^2 V_{total}}{\partial \phi_j \partial \phi_i} \delta\phi_i - \kappa \dot{\phi}_j = 0, \quad (22)$$

where a is the scale factor, $\phi_{1(2)} = \sigma(\phi)$, \mathcal{R} is the curvature perturbation, $\Psi \equiv -\sum_j \dot{\phi}_j \delta\phi_j$ is the total momentum current potential, p and ρ are the total pressure and energy density, and $\omega \equiv \frac{p}{\rho}$. The gauge-invariant curvature perturbation Φ_H is given by

$$\Phi_H = 4\pi G \frac{a^2}{k^2} \delta\rho_{com}. \quad (23)$$

²As mentioned in [8], it is not so obvious how to distribute the constant V_0 into $V(\sigma)$ and $V(\phi)$ in Eq. (13). We have checked Eq. (13) by numerical calculation as shown in Fig.1.

We have found that the above analytic solution Eq.(15) agrees well with the numerical results. These two primordial spectra are plotted in Fig.1. From Fig.1, one can see that the analytic solution (the solid line) agrees with the numerical results very well, which support the validity of the approximation used above.

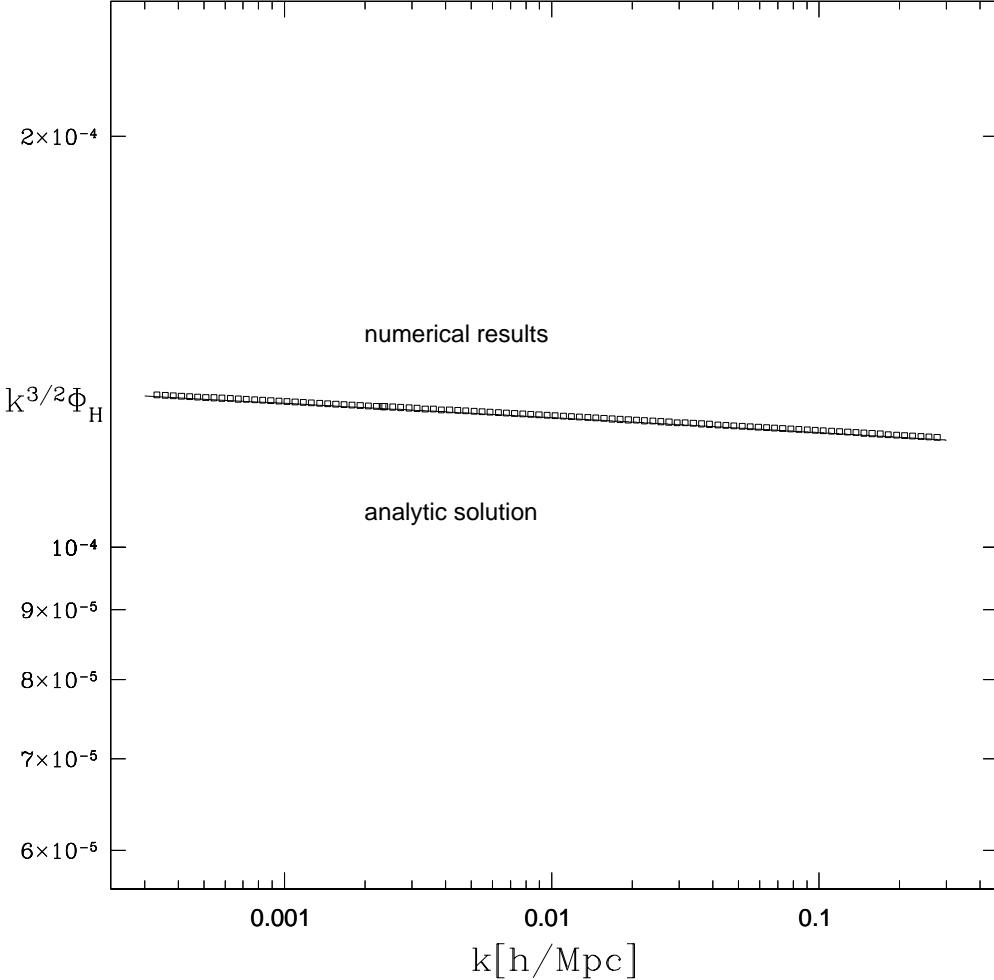


Figure 1: The analytic and numerical results for the primordial spectra. We have taken $d = 6$, $g = \lambda = \kappa = 0.1$, $\sigma_i = 0.114M$, $\phi_i = 0.0278M$ in numerical calculation.

The spectral index is given by

$$n = 1 + 2 \frac{d \ln k^{3/2} \Phi_H(k)}{d \ln k}. \quad (24)$$

Substituting Eq.(12) (or Eq.(15)) into Eq. (24), the spectral index n_{COBE} on COBE

scales

$$n_{\text{COBE}} \simeq 1 - 3M^2 \frac{V'(\sigma)^2}{V_0^2} + 2M^2 \frac{V''(\sigma)}{V_0} - M^2 \frac{V'(\phi)^2}{V_0^2}, \quad (25)$$

$$\simeq 1 + 2M^2 \frac{V''(\sigma)}{V_0}, \quad (26)$$

$$= 1 - \frac{1}{N_{\text{COBE}}} \simeq 0.98. \quad (27)$$

We have also obtained $n_{\text{COBE}} = 0.98$ from our numerical calculation. Note that $n_{\text{COBE}} = 1.2 \pm 0.3$ is implied by present CMB observations and hence the CMB observation does not restrict any parameters in this model as opposed to the result in Ref. [2]. This discrepancy is due to the incorrect estimation of the gauge invariant ζ in Ref. [2] where ζ is given by

$$\zeta = \frac{\delta\rho}{\rho + p} \propto \frac{V'(\phi) + V'(\sigma)}{V'(\phi)^2 + V'(\sigma)^2} \delta\phi. \quad (28)$$

On the other hand, the accurate form of ζ is given by

$$\zeta = \frac{\Delta_g}{1 + \omega} - \frac{16\pi G a^2 p}{(1 + \omega) k^2} \Pi, \quad (29)$$

where $\Delta_g \equiv \delta + 3(1 + \omega)\mathcal{R}$, $\delta \equiv \delta\rho/\rho$, and Π is an anisotropic stress perturbation. Hence only if we take the flat slicing and an anisotropic stress perturbation can be neglected, then the first equality in Eq. (28) is satisfied. It is also worth noting that the dynamics of the perturbed system cannot be described by one equation for ζ when more than one scalar field are involved [9, 10]. In addition, the expression (28) is based on $\delta\sigma = \delta\phi$. However, $\delta\sigma$ and $\delta\phi$ are classical random quantities, and the correct expression is $\langle (\delta\sigma)^2 \rangle = \langle (\delta\phi)^2 \rangle$, where $\langle \dots \rangle$ means ensemble average. The results of numerical calculation for the time evolutions of ζ and $\frac{V'(\phi) + V'(\sigma)}{V'(\phi)^2 + V'(\sigma)^2} \delta\phi$ during inflation are plotted in Fig.2. As seen in Fig.2, the two quantities are not proportional to each other.

4 Isocurvature Fluctuations

During inflation, the fluctuation of the phase θ of the AD field is given by

$$\delta\theta = \frac{H}{\sqrt{2k^3\phi}}. \quad (30)$$

After baryogenesis by the AD mechanism, the fluctuation of θ becomes baryonic isocurvature perturbation [11]. According to Ref. [2], the baryon number density n_B

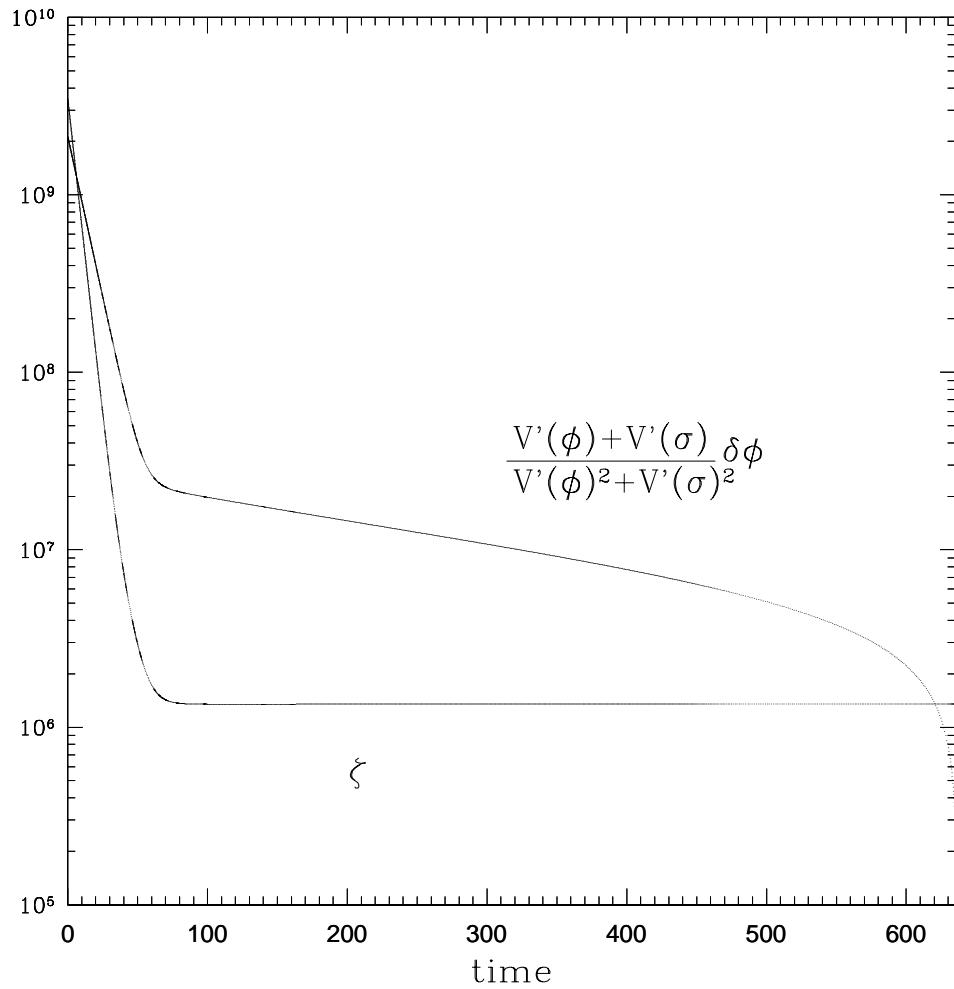


Figure 2: The time evolutions of the gauge invariant ζ and $\frac{V'(\phi) + V'(\sigma)}{V'(\phi)^2 + V'(\sigma)^2} \delta\phi$ during inflation.

is proportional to $\sin 2\theta$. Thus the isocurvature fluctuation with comoving wavenumber k is given by

$$\delta_{iso}(k) = \frac{\delta n_B^{iso}}{n_B} \frac{\Omega_B}{\Omega_t}, \quad (31)$$

$$= 2 \cot 2\theta_k \frac{H(t_k)}{\sqrt{2k^3 \phi(t_k)}} \frac{\Omega_B}{\Omega_t}, \quad (32)$$

where $\Omega_{B(t)}$ is the density parameter of baryons (total matter), t_k is the time when the given mode crosses the Hubble radius during inflation. On the other hand, both the AD field and the inflaton generate the adiabatic fluctuation given by

$$\delta_{ad}(k) = \frac{2}{3} \left(\frac{k}{a(t)H(t)} \right)^2 \Phi_H \quad (33)$$

$$= \frac{2}{3} \left(\frac{k}{a(t)H(t)} \right)^2 \times \frac{2}{3} \frac{1}{M^2} \left(\frac{V(\sigma)}{V'(\sigma)} \delta\sigma + \frac{V(\phi)}{V'(\phi)} \delta\phi \right) \quad (34)$$

$$= \frac{2\sqrt{2}}{9} \frac{\sqrt{k}H(t_k)}{a(t)^2 H(t)^2 M^2} \left(\frac{8\pi^2}{g^2} \sigma(t_k) + \frac{\phi(t_k)}{2d-2} \right), \quad (35)$$

where t is an arbitrary time. To compare these two types of fluctuations, it may be natural to consider the ratio of the power spectra at horizon crossing, i.e., $k^{-1}a(t) = H(t)^{-1}$, which is written as [12]

$$\alpha_{KSY} = \left. \frac{P_{iso}}{P_{ad}} \right|_{k=a(t)H(t)} = \frac{81g^4 M^4}{256\pi^4 \phi(t_k)^2 \sigma(t_k)^2} \left(\frac{\Omega_B}{\Omega_t} \right)^2 \cot^2 2\theta_k. \quad (36)$$

It is also useful to consider the ratio α of the present power spectra [13]. The present power spectrum can be described as

$$P(k) = P_{ad} + P_{iso} = A_{ad} k T_{ad}^2(k) + A_{iso} k T_{iso}^2(k), \quad (37)$$

where the transfer functions ($T_{ad, iso}$) are normalized as $T(k \rightarrow 0) = 1$. The ratio α is defined as

$$\alpha \equiv \frac{A_{iso}}{A_{ad}}. \quad (38)$$

Actually, α is related to α_{KSY} [13] by $\alpha = \left(\frac{4}{27} \right)^2 \alpha_{KSY}$. From Eqs. (5), (11), and (36), we obtain a lower limit for α as

$$\alpha = \frac{g^4 M^4}{144\pi^4 \phi(t_k)^2 \sigma(t_k)^2} \cot^2 2\theta_k \left(\frac{\Omega_B}{\Omega_t} \right)^2, \quad (39)$$

$$> \alpha_c, \quad (40)$$

where α_c is given by

$$\alpha_c = \begin{cases} \frac{\sqrt{30}}{72\pi^2} g \lambda \cot^2 2\theta_k \left(\frac{\xi}{M}\right)^{-2} N_k^{-1} \left(\frac{\Omega_B}{\Omega_t}\right)^2 & d = 4 \\ \left(\frac{5}{72}\right)^{\frac{1}{4}} \frac{g^{\frac{3}{2}} \lambda^{\frac{1}{2}}}{12\pi^2} \cot^2 2\theta_k \left(\frac{\xi}{M}\right)^{-1} N_k^{-1} \left(\frac{\Omega_B}{\Omega_t}\right)^2 & d = 6 \end{cases}. \quad (41)$$

If we take $N = 50$, $\Omega_B = 0.03h^{-2}$, $\Omega_t = 0.28$, and $h = 0.8$,³ this lower limit is approximately given by

$$\alpha_c \simeq \begin{cases} 0.52g\lambda \cot^2 2\theta_k & d = 4 \\ 8.4 \times 10^{-4} g^{\frac{3}{2}} \lambda^{\frac{1}{2}} \cot^2 2\theta_k & d = 6 \end{cases}. \quad (42)$$

If $g \simeq \lambda \simeq 0.1$ and $\theta \sim O(1)$, then $\alpha_c \sim 1.1 \times 10^{-3}$ for the $d = 4$ case. According to [14], the best-fit value⁴ of α is $\alpha_{best-fit} = 2.4 \times 10^{-3}$. Finally, we have constraints on g , λ , and θ :

$$g\lambda \cot^2 2\theta_k < 4.7 \times 10^{-3}, \quad d = 4, \quad (43)$$

$$g^{\frac{3}{2}} \lambda^{\frac{1}{2}} \cot^2 2\theta_k < 2.9, \quad d = 6. \quad (44)$$

Thus, the large coupling constants can be excluded for $d = 4$. On the other hand, the constraint for $d = 6$ (and $d \geq 8$) is not severe at all. It is noticed that the value of the AD field during inflation can be smaller than that used here for some initial conditions because the AD field may oscillate before inflation. For this case, the constraint becomes more stringent.

5 Conclusion

In this paper we have considered the adiabatic and isocurvature fluctuations of the AD field in the D-term inflation model, and have found that there exists an upper limit for AD field due to its dynamical property and the requirement that an inflation should occur. The primordial spectrum has been calculated analytically and numerically, and has been found to be of the familiar Harrison-Zeldovich type. While the adiabatic fluctuations of the AD field do not make any significant contribution, the isocurvature fluctuations of the AD field can generate baryonic isocurvature perturbations. The upper bound for the AD field in turn leads to the lower limit for

³These are best-fit values of model(10) in [14]. The fit was done for the data from Boomerang and MAXIMA-1.

⁴ α_{EKV} defined as Eq.(6) in [14] is related to our definition of α as follows.

$$\alpha = \frac{\alpha_{EKV}}{36(1 - \alpha_{EKV})}.$$

We adopt the model (10) in [14], because both adiabatic and isocurvature perturbations are almost scale-invariant in our model.

isocurvature fluctuation as Eq.(42). Taking account of the observational constraints on isocurvature perturbations from Boomerang and MAXIMA-1, we had interesting constraints on some combinations of g , λ , and θ , especially in the case of $d = 4$.

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